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Polynomial differential systems with even degree have no global centers

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ABSTRACT

Let $\dot{x} = P(x,y)$, $\dot{y} = Q(x,y)$ be a differential system with P and Q real polynomials, and let $d = \max\{\deg P, \deg Q\}$. A singular point p of this differential system is a global center if $\mathbb{R}^2 \setminus \{p\}$ is filled with periodic orbits. We prove that if d is even then the polynomial differential systems have no global centers.

1. Introduction and statement of the main results

A singular point q of a vector field defined in $\mathbb{R}^2$ is a *center* if it has a neighborhood filled of periodic orbits with the unique exception of q. The *period annulus* of the center q is the maximal neighborhood U of q such that all the orbits contained in U are periodic except of course q. A center is *global* if its period annulus is $\mathbb{R}^2 \setminus \{q\}$. The notion of center goes back to Poincaré, see [7].

It is well known that any quadratic polynomial system (i.e. $n = 2$) has no global centers. The proof of this result is very large. It is based in classifying all the centers of the quadratic systems and then see that they are not global centers, see [1–3,8,9].

Let $P, Q \in \mathbb{R}[x,y]$ and $d = \max\{\deg P, \deg Q\}$. We will show that the polynomial differential system

$$\dot{x} = P(x,y), \quad \dot{y} = Q(x,y), \tag{1}$$

with d even do not have global centers. This is the main aim of this paper. Our proof for all d even is shorter than the existing one for $d = 2$.

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