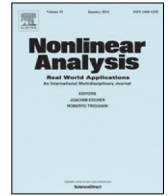




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## Nonlinear Analysis: Real World Applications

[www.elsevier.com/locate/nonrwa](http://www.elsevier.com/locate/nonrwa)
A sufficient condition for the real Jacobian conjecture in  $\mathbb{R}^2$ Jaume Llibre<sup>a</sup>, Claudia Valls<sup>b,\*</sup><sup>a</sup> *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*<sup>b</sup> *Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1049-001, Lisboa, Portugal*

## ARTICLE INFO

*Article history:*

Received 12 February 2020

Received in revised form 29 January 2021

Accepted 2 February 2021

Available online xxxx

*Keywords:*Real Jacobian conjecture  
Global injectivity  
Center

## ABSTRACT

Let  $F = (f, g): \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a polynomial map such that  $\det DF(x, y)$  is different from zero for all  $(x, y) \in \mathbb{R}^2$ . We provide some new sufficient conditions for the injectivity of  $F$ . The proofs are based on the qualitative theory of differential equations.

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## 1. Introduction and statement of the main result

Let  $F = (f, g): \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a smooth map such that  $\det DF(x, y)$  is different from zero for all  $(x, y) \in \mathbb{R}^2$ . By the Inverse Function Theorem, it is clear that  $F$  is a local diffeomorphism, but it is not always injective. There are very general well known additional conditions to ensure that  $F$  is a global diffeomorphism, see for instance [1–3].

If  $F$  is a polynomial map, the statement that  $F$  is injective is known as the *real Jacobian conjecture*. This conjecture is false, because Pinchuk constructed, in [4], a non-injective polynomial map with nonvanishing Jacobian determinant. Thus it is natural to ask for additional conditions in order that this conjecture holds. In [5,6], for instance, it was shown that for the injectivity of  $F$  it is enough to assume that the degree of  $f$  is less than or equal to 4. If we assume that  $\det DF(x)$  is a constant different from zero, then to know if  $F$  is injective is an open problem largely known as the *Jacobian conjecture* (see [7] and [8] for details and for surveys on the Jacobian conjecture and related problems). In [9] the authors provide a sufficient condition for the validity of the real Jacobian conjecture. More precisely they proved the following theorem.

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