Nonlinear Analysis: Real World Applications 60 (2021) 103298

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

A sufficient condition for the real Jacobian conjecture in $\mathbb{R}^2$

Jaume Llibre [a], Claudia Valls [b,*]

[a] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*

[b] *Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1049–001, Lisboa, Portugal*

ARTICLE INFO

Article history:
Received 12 February 2020
Received in revised form 29 January 2021
Accepted 2 February 2021
Available online xxxx

Keywords:
Real Jacobian conjecture
Global injectivity
Center

ABSTRACT

Let $F = (f, g)\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that $\det DF(x, y)$ is different from zero for all $(x, y) \in \mathbb{R}^2$. We provide some new sufficient conditions for the injectivity of F. The proofs are based on the qualitative theory of differential equations.

© 2021 Elsevier Ltd. All rights reserved.

1. Introduction and statement of the main result

Let $F = (f, g)\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a smooth map such that $\det DF(x, y)$ is different from zero for all $(x, y) \in \mathbb{R}^2$. By the Inverse Function Theorem, it is clear that F is a local diffeomorphism, but it is not always injective. There are very general well known additional conditions to ensure that F is a global diffeomorphism, see for instance [1–3].

If F is a polynomial map, the statement that F is injective is known as the *real Jacobian conjecture*. This conjecture is false, because Pinchuk constructed, in [4], a non-injective polynomial map with nonvanishing Jacobian determinant. Thus it is natural to ask for additional conditions in order that this conjecture holds. In [5,6], for instance, it was shown that for the injectivity of F it is enough to assume that the degree of f is less than or equal to 4. If we assume that $\det DF(x)$ is a constant different from zero, then to know if F is injective is an open problem largely known as the *Jacobian conjecture* (see [7] and [8] for details and for surveys on the Jacobian conjecture and related problems). In [9] the authors provide a sufficient condition for the validity of the real Jacobian conjecture. More precisely they proved the following theorem.

* Corresponding author.
E-mail addresses: jllibre@mat.uab.cat (J. Llibre), cvalls@math.ist.utl.pt (C. Valls).

https://doi.org/10.1016/j.nonrwa.2021.103298
1468-1218/© 2021 Elsevier Ltd. All rights reserved.