PERIODIC ORBITS FOR A GENERALIZED HÉNON-HEILES HAMILTONIAN SYSTEM WITH AN ADDITIONAL SINGULAR GRAVITATIONAL TERM

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Abstract. Using the averaging theory of first order we study analytically the existence of two families of periodic orbits of a generalized Hénon-Heiles Hamiltonian system. Moreover we characterize when this generalized Hénon-Heiles Hamiltonian system has or has not a second C^1 first integral independent with the Hamiltonian.

1. Introduction and statement of results

The classical Hénon-Heiles Hamiltonian

$$H = \frac{1}{2}(p_x^2 + p_y^2 + x^2 + y^2) + x^2 y - \frac{y^3}{3}.$$

was introduced in 1964 as a model for studying the existence of a third integral of motion of a star in an rotating meridian plane of a galaxy in the neighborhood of a circular orbit [14] and it becomes a paradigm for nonlinear dynamics of Hamiltonian systems.

In this paper we study the generalized Hénon-Heiles Hamiltonian system with an additional singular gravitational term of the form

$$H_\varepsilon = \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(x^2 + y^2) + x^2 y - \frac{y^3}{3} - \varepsilon^5 \frac{1}{x^2 + y^2}, \tag{1}$$

where $\varepsilon \geq 0$ is a small parameter (note that when $\varepsilon = 0$ the Hamiltonian H_0 is the classical Hénon-Heiles Hamiltonian).

We study the periodic dynamics of the Hénon-Heiles Hamiltonian system with the additional singular gravitational term $1/(x^2 + y^2)$. The Hénon-Heiles modelizes how stars move around a galactic center. The addition of this singular gravitational term allows to modelize the motion of the stars in a pseudo or post-Newtonian dynamics. Thus this model allows to predict phenomena which cannot be detected by the classical Newtonian mechanics.

Other generalizations of the Hénon-Heiles Hamiltonian system (1) with different additional singular gravitational terms was introduced in [20] where the authors classify numerically sets of starting conditions for the trajectories. The additional singular gravitational term in our system provides more accurate and realistic dynamics of a test particle moving in the central region of a galaxy and creates a

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