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THE EULER-JACOBI FORMULA AND THE PLANAR QUADRATIC-QUARTIC POLYNOMIAL DIFFERENTIAL SYSTEMS

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ABSTRACT. The Euler-Jacobi formula provides an algebraic relation between the singular points of a polynomial vector field and their topological indices. Using this formula we obtain the configuration of the singular points together with their topological indices for the planar quadratic-quartic polynomial differential systems when these systems have eight finite singular points.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

Consider the planar polynomial differential system

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

in $\mathbb{R}^2$ where $P(x, y)$ and $Q(x, y)$ are real polynomials of degree n and m, respectively. Assuming that system (1) has nm finite singular points using the Euler-Jacobi formula we obtain an algebraic relation between the finite singular points of the polynomial differential system (1) and the topological indices of these finite singular points. A proof of the Euler-Jacobi formula can be found in [1].

It also follows from Bezout's Theorem that in the complex projective plane, and taking into account all the multiplicities of the singular points, if the number of singular points is finite, then it is at most nm. The Euler-Jacobi formula deals with the case in which all the singular points have multiplicity one and are located in the finite part of the projective space. In the two-dimensional case this formula can be enunciated as follows. Consider a system of two real polynomials of degrees n and m, respectively, in the variables x and y. If the set of zeroes of that system (that we denote by A) contains exactly nm elements, then the Jacobian determinant

$$J = \det \begin{pmatrix} \partial P/\partial x & \partial P/\partial y \\ \partial Q/\partial x & \partial Q/\partial y \end{pmatrix}$$

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