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Generalized Analytic Integrability of a Class of Polynomial Differential Systems in $\mathbb{C}^2$

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Abstract
This paper study the type of integrability of differential systems with separable variables $\dot{x} = h(x) f(y)$, $\dot{y} = g(y)$, where h, f and g are polynomials. We provide a criterion for the existence of generalized analytic first integrals of such differential systems. Moreover we characterize the polynomial integrability of all such systems.

In the particular case $h(x) = (ax + b)^m$ we provide necessary and sufficient conditions in order that this subclass of systems has a generalized analytic first integral. These results extend known results from Giné et al. (Discrete Contin. Dyn. Syst. 33:4531–4547, 2013) and Llibre and Valls (Discrete Contin. Dyn. Syst., Ser. B 20:2657–2661, 2015). Such differential systems of separable variables are important due to the fact that after a blow-up change of variables any planar quasi-homogeneous polynomial differential system can be transformed into a special differential system of separable variables $\dot{x} = xf(y)$, $\dot{y} = g(y)$, with f and g polynomials.

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1 Introduction and the main results

Planar polynomial differential systems play an important role in the qualitative theory of dynamical systems due to their many applications in physics, chemist, biology, economics, ... Nowadays the qualitative theory has gained wide development for polynomial systems. For a planar differential system, the existence of a first integral determines completely its global dynamical behavior. So a natural problem arises: Given a polynomial differential system in

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