GLOBAL FLOW OF THE ROTATING KEPLER PROBLEM

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Abstract. A complete description of the global flow of the rotating Kepler problem is given by means of some invariant sets that, in contrast to the usual Jacobi levels, can be embedded in $\mathbb{R}^3$. These sets are constructed by adjoining to the Jacobi levels two invariant boundary manifolds representing motion at collision and motion at infinity, and which are obtained through the use of the blowing up techniques. The foliation of the phase space into invariant sets and the flow on them is shown for all values of the Jacobi constant.

1. INTRODUCTION AND DEFINITIONS

We consider the circular planar restricted three-body problem in a rotating coordinate system (q_1, q_2) of rotational frequency unity, called the synodic coordinate system. The larger primary P_1 of mass $1-\mu$ is located at the origin and the smaller primary P_2 of mass μ at position $e_2 = (-1, 0)$. Then, the Hamiltonian for the motion of an infinitesimal body P_3 is given by

$$H = \frac{1}{2}\|p\|^2 + p^t Jq - \|q\|^{-1} + \mu(-\|q\|^{-1} - \|q-e_2\|^{-1} - p_2),$$

where (p_1, p_2) are the conjugate momenta corresponding to variables (q_1, q_2),

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

and $\|\|$ is the Euclidean norm in $\mathbb{R}^2$. The function $C = -2H$ is a first integral of the system, usually called the *Jacobi integral.* Notice that the Jacobi constant differs from the usual one in the quantity $\mu(1-\mu)$ (see [**Sz**]).

When the mass parameter μ is zero, the problem reduces to a Kepler problem in a uniformly rotating frame of reference. This problem is usually called the *rotating Kepler problem* and is defined by the Hamiltonian

$$H = \frac{1}{2}\|p\|^2 + p^t Jq - \|q\|^{-1},$$