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Periodic structure of the transversal maps on surfaces

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Abstract. In this article we study the set of periods of transversal maps on orientable and non-orientable compact surfaces without boundary. We provide sufficient conditions, in terms of the spectra of the induced maps on homology, in order that the map has infinitely many periods, in particular odd periods.

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1. Introduction and statements of the main results

Let f be a continuous self-map on X. If $x \in X$ and $f(x) = x$ we say that x is a *fixed point* of the map f. If $f^n(x) = x$ and $f^k(x) \neq x$ for all $k = 1, \ldots, n-1$, then we say that x is a *periodic point* of the map f of *period* n. We denote by Per(f) the set of the periods of all periodic points of a map $f : X \to X$.

Let X be a n-dimensional topological manifold and f a continuous self-map on X. The map f induces a homomorphism on the k-th rational homology group of X for $0 \leq k \leq n$, i.e. $f_{*k} : H_k(X, \mathbb{Q}) \to H_k(X, \mathbb{Q})$. The $H_k(X, \mathbb{Q})$ is a finite dimensional vector space over $\mathbb{Q}$ and f_{*k} is a linear map whose matrix has integer entries.

The *Lefschetz number* of the map f is an integer defined as

$$L(f) = \sum_{k=0}^{n} (-1)^k \text{trace}(f_{*k}). \tag{1}$$

The *Lefschetz Fixed Point Theorem* states that if $L(f) \neq 0$ then f has a fixed point (*cf.* [2] or [13]).

The *Lefschetz numbers of period* m are defined by

$$\ell(f^m) := \sum_{r|m} \mu(r) L(f^{m/r}), \tag{2}$$

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