JOURNAL OF DIFFERENTIAL EQUATIONS 37, 444-465 (1980)

Some Homoclinic Phenomena in the Three-Body Problem

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Received October 4, 1979

We prove the existence of transversal homoclinic points in the collinear three-body problem, restricted and general, and in the planar circular restricted three-body problem. As a consequence the shift of Bernoulli is proved to be included as a subsystem of a suitable section of the flow for the three cases studied. Then the existence of all the possible types of final evolution follows.

1. Introduction and Main Results

In 1960 Sitnikov [9] gave a proof of the existence of oscillatory solutions in the three-body problem. For such solutions $\underline{\lim}(\sup(r_{ij}))$ is bounded but $\overline{\lim}(\sup(r_{ij}))$ is not. Oscillatory motions were predicted by Chazy as early as 1929 [2]. The results of Sitnikov were made rigourous and widely extended by Alekseev in 1967 [1]. The machinery is quite impressive but Moser [7] was able to do things in a more tractable way.

What is actually proved in the Sitnikov problem (two equal masses describing a Keplerian bounded orbit in a plane and a third infinitesimal one in the line orthogonal to the plane through the center of the two finite masses) is that near the parabolic orbits of the third body the Bernoulli shift based on countable many symbols can be included as a subsystem of a Poincaré map associated to the flow. This implies not only the existence of oscillatory solutions but that of an infinity of periodic, capture and escape orbits. A basic fact used to prove the embedding of the shift is the existence of transversal homoclinic points.

The purpose of this paper is to give different examples of transversal homoclinic phenomena in the three-body problem. First we consider a restricted collinear problem. Let $m_1 = m$, $m_2 = 1 - m$, $-x_1$, x_2 be the masses and positions of the primaries and x the position of the infinitesimal body. We

0022-0396/80/090444-22$02.00/0