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Oscillatory Solutions in the Planar Restricted Three-Body Problem

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1. Introduction

In this paper we prove the existence of oscillatory solutions in the circular planar restricted three-body problem. Let m_1, m_2 be the masses of the primaries normalized in such a way that $m_1 = 1-m$, $m_2 = m$, $m \in (0,1)$. Units of length and time are chosen in order to have one unit of distance between the primaries and a mean motion equal to 1.

For the position of the infinitesimal body m_3 we use both the sidereal coordinates (X, Y) and the synodical ones (x, y). In the last system the two primaries are fixed at $(m, 0)$, $(m-1, 0)$, respectively. Then the equations of motion are (see Szebehely [7]):

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}, \qquad \ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y}, \tag{1.1}$$

where $\Omega(x, y) = (x^2 + y^2)/2 + (1-m)/r_1 + m/r_2 + m(1-m)/2$, and $r_1^2 = (x-m)^2 + y^2$, $r_2^2 = (x+1-m)^2 + y^2$.

System (1.1) has the Jacobi first integral

$$C = C(x, y, \dot{x}, \dot{y}) = 2\Omega(x, y) - (\dot{x}^2 + \dot{y}^2) \tag{1.2}$$

which equals two times the difference between the angular momentum with respect to the origin, M, and the energy, h, both in sidereal coordinates.

For large values of C the zero velocity curves defined by $2\Omega(x, y) - C = 0$ have three components. We only consider motion in the unbounded component R of the admissible region $2\Omega(x, y) \geq C$. Let r be the distance from m_3 to the origin. An oscillatory solution is characterized by the fact that $\limsup_{t \to +\infty} r(t) = +\infty$, but $\liminf_{t \to +\infty} r(t) < +\infty$. We prove the existence of such orbits by the usual method of symbolic dynamics. Furthermore we prove the existence of all possible types of final evolution.

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