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On the Hénon-Heiles potential

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Abstract. The epoch making paper by Hénon and Heiles [10] renewed the interest in nonintegrable hamiltonian systems. Recent contributions to this problem can be found in [3], [11], [5]. This communication states new results on periodic orbits with an analysis of the stable or unstable character and transitions of stability, and on the integrability of the system. Several numerical computations are included. Full details and proofs will appear elsewhere.

§1. Introduction. Looking for a third integral [7], [20] for the motion of a particle in a galactic potential with cylindrical symmetry Hénon and Heiles (heretofore called HH) [10] were leaded to the hamiltonian

$$H(x,y) = \frac{1}{2}(y_1^2+y_2^2) + \frac{1}{2}(x_1^2+x_2^2) + \frac{1}{3}x_1^3 - x_1x_2^2$$

where the position is given by $x=(x_1,x_2)$ and the momentum by $y=(y_1,y_2)$. The hamiltonian equations $\dot{x}=H_y$, $\dot{y}=-H_x$ are extremely easy:

$$\dot{x}_1 = y_1, \quad \dot{x}_2 = y_2, \quad \dot{y}_1 = -x_1 - x_1^2 + x_2^2, \quad \dot{y}_2 = -x_2 + 2x_1x_2 .$$

However the solutions are not easy at all and the motion is interesting enough as you can verify if you look for the remaining of the paper.

The techniques used for this hamiltonian and that allow us to obtain the results can be applied to a large class of hamiltonian systems with $n=2$ degrees of freedom, specially to the following ones: