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ON A CLASS OF POLYNOMIAL DIFFERENTIAL SYSTEMS OF DEGREE 4: PHASE PORTRAITS AND LIMIT CYCLES

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Abstract. In this paper we characterize the phase portraits in the Poincaré disc of the class of polynomial differential systems of the form

$$\dot{x} = -y, \qquad \dot{y} = x + ax^4 + bx^2y^2 + cy^4,$$

with $a^2+b^2+c^2 \neq 0$, which are symmetric with respect to the x-axis. Such systems have a center at the origin of coordinates. Moreover, using the averaging theory of five order, we study the number of limit cycles which can bifurcate from the period annulus of this center when it is perturbed inside the class of all polynomial differential systems of degree 4.

1. Introduction and statement of the main results

By definition a *polynomial differential system* in $\mathbb{R}^2$ is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x,y), \qquad \frac{dy}{dt} = \dot{y} = Q(x,y), \tag{1.1}$$

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