Journal of Geometry and Physics 167 (2021) 104290

Contents lists available at ScienceDirect

Journal of Geometry and Physics

www.elsevier.com/locate/geomphys

Periodic orbits and equilibria for a seventh-order generalized Hénon-Heiles Hamiltonian system

Jaume Llibre [a], Tareq Saeed [b], Euaggelos E. Zotos [c,*]

[a] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
[b] Nonlinear Analysis and Applied Mathematics (NAAM)-Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia
[c] Department of Physics, School of Science, Aristotle University of Thessaloniki, GR-541 24, Thessaloniki, Greece

ARTICLE INFO

Article history:
Received 31 August 2020
Received in revised form 9 May 2021
Accepted 17 May 2021
Available online 21 May 2021

Keywords:
Eneralized Hénon-Heiles potential
Finite equilibria
Infinite equilibria

ABSTRACT

In this paper we study analytically the existence of two families of periodic orbits using the averaging theory of second order, and the finite and infinite equilibria of a generalized Hénon-Heiles Hamiltonian system which includes the classical Hénon-Heiles Hamiltonian. Moreover we show that this generalized Hénon-Heiles Hamiltonian system is not C^1 integrable in the sense of Liouville–Arnol'd, i.e. it has not a second C^1 first integral independent with the Hamiltonian. The techniques that we use for obtaining analytically the periodic orbits and the non C^1 Liouville-Arnol'd integrability, can be applied to Hamiltonian systems with an arbitrary number of degrees of freedom.

1. Introduction and statement of results

The classical Hénon-Heiles Hamiltonian consists of a two dimensional harmonic potential plus two cubic terms, i.e.

$$H = \frac{1}{2}(p_x^2 + p_y^2 + x^2 + y^2) + x^2 y - \frac{y^3}{3}.$$

This Hamiltonian was introduced in 1964, it is a model for studying the existence of a third integral of motion of a star in an rotating meridian plane of a galaxy in the neighborhood of a circular orbit [8].

The generalized Hénon-Heiles Hamiltonian system here studied is

$$\begin{aligned} H_\varepsilon = \ & \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(x^2 + y^2) + x^2 y - \frac{y^3}{3} + \varepsilon\Big(x^6 y + x^4 y^3 + x^4 y \\ & + x^2 y^5 + x^2 y^3 - \frac{y^7}{7} - \frac{y^5}{5} + \frac{1}{4}(x^2 + y^2)^2 + \frac{1}{6}(x^2 + y^2)^3\Big), \end{aligned} \tag{1}$$

where $\varepsilon \geq 0$ is a small parameter. Of course, when $\varepsilon = 0$ the Hamiltonian H_0 is the classical Hénon-Heiles Hamiltonian. The Hamiltonian (1) was introduced in [7].

* Corresponding author.
E-mail addresses: jllibre@mat.uab.cat (J. Llibre), tsalmalki@kau.edu.sa (T. Saeed), evzotos@physics.auth.gr (E.E. Zotos).

https://doi.org/10.1016/j.geomphys.2021.104290
0393-0440/