ON THE NUMBER OF FIXED POINTS FOR A CONTINUOUS MAP OF A FINITE CONNECTED GRAPH

by

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ABSTRACT.

Let F_n be a bouquet of n circles. For an arbitrary continuous map f: $F_n \to F_n$ we shall define a non-negative integer m (f), easily computable in terms of the induced homomorphism $f_*: \pi_1 (F_n) \to \pi_1 (F_n)$. This integer is the best lower bound of the number of fixed points for the homotopy class of f. This result generalizes the well known fact that a continuous map f of the circle into itself has at least m (f) = |1 − degree (f)| fixed points.

§ 1. INTRODUCTION.

Let F_n be a bouquet of n circles, that is, the quotient space of [0,n] obtained by identifying all points of integer coordinates to a single point p.

This paper is related with the following question. If f: $F_n \to F_n$ is a continuous map, what can be said about the number of fixed points of f? Theorem A gives a complete answer for all maps homotopic to f rather than just for the map f itself, as is usual in fixed point theory (see [1]). In fact, we generalize to a bouquet of circles the well known result ([1,p 107]) that a continuous map f of the circle into intself has at least |1 − degree (f)| fixed points.

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$$f_*: \pi_1 (F_n) \to \pi_1 (F_n) \text{ (see §2).}$$

Let N(f) be the Nielsen number of the map f (see [1] or §4). Our main results are the following.