# ON THE ELLIPTIC RESTRICTED THREE-BODY PROBLEM

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**Abstract.** The main goal of this paper is to show that the elliptic restricted three-body problem has ejection–collision orbits when the mass parameter $\mu$ is small enough. We make use of the 'blow up' techniques. Moreover, we describe the global flow of the elliptic problem when $\mu = 0$ taking into account the singularities due to collision and to infinity.

## 1. Introduction

Let $P_1$, $P_2$ and $P_3$ be three bodies moving in the space $\mathbf{R}^3$ according to Newton's law of gravitation. The masses of the primaries $P_1$ and $P_2$ are $m_1 = 1 - \mu$ and $m_2 = \mu$ respectively, with $\mu \in [0, 1)$. The parameter $\mu$ is called the mass parameter. The third body has infinitesimal mass. We assume $P_2$ describes an elliptic orbit around $P_1$ with eccentricity $e$, true anomaly $f$ and semimajor axis $a = 1$.

Taking $f$ as independent variable, the equations of motion for the infinitesimal mass in a rotating and pulsating coordinates $(\bar\rho, \bar\eta, \bar\varphi)$ (see [17] pp. 590–594 and Section 2) are

$$\begin{aligned} \bar\rho'' - 2\bar\eta' &= \omega_{\bar\rho}, \\ \bar\eta'' + 2\bar\rho' &= \omega_{\bar\eta}, \\ \bar\varphi'' + \bar\varphi &= \omega_{\bar\varphi}, \end{aligned} \tag{1.1}$$

where prime indicates d/df, and

$$\omega = (1 + e\cos f)^{-1}\Omega,$$

$$\Omega = \tfrac{1}{2}(\bar\rho^2 + \bar\eta^2 + \bar\varphi^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \tfrac{1}{2}\mu(1-\mu),$$

$$r_1^2 = (\bar\rho - \mu)^2 + \bar\eta^2 + \bar\varphi^2,$$

$$r_2^2 = (\bar\rho - \mu + 1)^2 + \bar\eta^2 + \bar\varphi^2.$$

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