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Polynomial Homogeneous Maps and Their Periods*

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Abstract We study the set of periods of the homogeneous polynomial maps $f : \mathbb{R}^n \to \mathbb{R}^n$ and $f : \mathbb{C}^n \to \mathbb{C}^n$ of degree $m > 1$. For these complex maps, we also describe the number of invariant straight lines through the origin by f^k for $k = 1, 2, \ldots$ and the dynamics of f^k over them.

Keywords Homogeneous polynomial map, Period, Set of periods, Self-maps of a sphere.

MSC(2010) 37F10, 37C25.

1. Introduction and statement of the main results

We consider discrete dynamical systems given by a real or complex homogeneous polynomial map defined in $\mathbb{R}^n$ or $\mathbb{C}^n$ respectively. For the discrete dynamical systems, the periodic orbits play an important role for understanding their dynamics. Perhaps, the best known example in this direction are the results contained in the paper entitle *"Period three implies chaos"* for continuous self–maps on the interval (see [21] or the book [2]).

The real homogeneous polynomial maps $f : \mathbb{R}^n \to \mathbb{R}^n$ have been studied by many authors (see, for instance, the survey of Aliashvili [1] and the references quoted therein). However, not too much attention has been paid to the study of their periodic orbits with the exception of their fixed points (see, for instance, [8,19,27]).

Let $\mathbb{CP}^n$ be the complex projective space of dimension n. The complex homogeneous polynomial maps $f : \mathbb{C}^n \to \mathbb{C}^n$ also has been considered in [1] and in the references quoted there, but again not too much attention was put into their periodic orbits. On the other hand, for the complex homogeneous polynomial maps $f : \mathbb{CP}^n \to \mathbb{CP}^n$, their set of periods have been studied (see Fornaes and Sibony [11], or [10]). On the other hand, these maps also have been studied from the point of view of their degree (see [17]).

The study of the set of periods of the real homogeneous polynomial maps $f : \mathbb{R}^n \to \mathbb{R}^n$ is our objective for such maps, while for complex homogeneous polynomial maps $f : \mathbb{C}^n \to \mathbb{C}^n$, our main goal is to the study their invariant straight lines

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