Euro. Jnl of Applied Mathematics (2021), *vol.* 32, *pp.* 317–336 © The Author(s), 2020. Published by Cambridge University Press. 317
doi:10.1017/S0956792520000145

On the birth and death of algebraic limit cycles in quadratic differential systems

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(*Received 20 December 2019*; *revised 2 April 2020*; *accepted 28 April 2020*; *first published online 26 May 2020*)

In 1958 started the study of the families of algebraic limit cycles in the class of planar quadratic polynomial differential systems. In the present we known one family of algebraic limit cycles of degree 2 and four families of algebraic limit cycles of degree 4, and that there are no limit cycles of degree 3. All the families of algebraic limit cycles of degree 2 and 4 are known, this is not the case for the families of degree higher than 4. We also know that there exist two families of algebraic limit cycles of degree 5 and one family of degree 6, but we do not know if these families are all the families of degree 5 and 6. Until today it is an open problem to know if there are algebraic limit cycles of degree higher than 6 inside the class of quadratic polynomial differential systems. Here we investigate the birth and death of all the known families of algebraic limit cycles of quadratic polynomial differential systems.

Key words: Algebraic limit cycles, Hopf bifurcation, homoclinic orbits, heteroclinic orbits, quadratic polynomial differential systems

2020 Mathematics Subject Classification: Primary: 34C07; 34C05; 37G15; 70K05

1 Introduction and statement of the main results

Consider the polynomial differential system in $\mathbb{R}^2$ given by

$$\begin{aligned} \dot{x} &= P(x,y), \\ \dot{y} &= Q(x,y), \end{aligned} \tag{1.1}$$

where the dot denotes derivative with respect to the *time* t. The number $m = \max\{\deg P, \deg Q\}$ is the *degree* of system (1.1).

When $m = 2$ system (1.1) is a *quadratic polynomial differential system* or simply a *quadratic system*. Associated with system (1.1) we have the polynomial vector field

$$\mathcal{X} = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}.$$