Discrete and Continuous Dynamical Systems - Series B
Vol. 27, No. 12, December 2022, pp. 7607–7623
doi:10.3934/dcdsb.2022056

PHASE PORTRAITS OF THE SELKOV MODEL IN THE POINCARÉ DISC

Jaume Llibre*

Departament de Matemàtiques, Facultat de Ciències
Universitat Autònoma de Barcelona, 08193 Bellaterra
Barcelona, Spain

Arefeh Nabavi

Department of Mathematical Sciences, Isfahan University of Technology
84156-83111, Isfahan, Iran

(Communicated by Pierre Magal)

Abstract. In this paper we classify the phase portraits in the Poincaré disc of the Selkov model for the glycolysis process

$$\dot{x} = -x + ay + x^2 y, \quad \dot{y} = b - ay - x^2 y,$$

in function of its parameters $a, b \in \mathbb{R}$. In particular we determine the regions in the parameter plane with biological meaning, i.e. with a, b, x and y positive.

1. **Introduction and statement of the main results.** Glycolysis is a process in which one molecule of glucose is broken down producing other molecules. This process is one of the most efficient processes in which the aerobic organisms oxidize nutrients to produce energy. In the glycolysis process some oscillations appear. These oscillations were first observed in 1957. Later, they have been observed in human muscle cells too like in cardiac muscle, skeletal muscle. In the 1960s extensive studies of these oscillations were done using many substrates and under various conditions. The period of oscillations is of the order of several minutes

In this paper we consider the Selkov model of glycolisis which is given by the following cubic differential system

$$\dot{x} = -x + ay + x^2 y, \quad \dot{y} = b - ay - x^2 y, \tag{1}$$

where x and y are the concentrations, and a and b are kinetic parameters. As usual the dot denotes derivative with respect to the time t. For more biological meanings about this system see the papers [1, 6, 12, 13, 14], and the references quoted there. For other related models see for instance [4].

Our purpose is to classify the global phase portraits of system (1) in the Poincaré disc, in function of its parameters, and determine the regions of the parameters with biological meaning.

2020 *Mathematics Subject Classification.* Primary 34C05; Secondary 34C07, 37C08.
Key words and phrases. Selkov model, Phase portrait, Poincaré compactification.
This work was partially supported by the Agencia Estatal de Investigación grants PID2019-104658GB-I00, and the H2020 European Research Council grant MSCA-RISE-2017-777911.
*Corresponding author: Jaume Llibre.