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Tranversality of the Invariant Manifolds Associated to the Lyapunov Family of Periodic Orbits near L_2 in the Restricted Three-Body Problem

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The restricted three-body problem is considered for values of the Jacobi constant C near the value C_2 associated to the Euler critical point L_2. A Lyapunov family of periodic orbits near L_2, the so-called family (c), is born for $C = C_2$ and exists for values of C less than C_2. These periodic orbits are hyperbolic. The corresponding invariant manifolds meet transversally along homoclinic orbits. In this paper the variation of the transversality is analyzed as a function of the Jacobi constant C and of the mass parameter μ. Asymptotical expressions of the invariant manifolds for $C \lesssim C_2$ and $\mu \gtrsim 0$ are found. Several numerical experiments provide accurate information for the manifolds and a good agreement is found with the asymptotical expressions. Symbolic dynamic techniques are used to show the existence of a large class of motions. In particular the existence of orbits passing in a random way (in a given sense) from the region near one primary to the region near the other is proved.

1. INTRODUCTION AND STATEMENT OF THE RESULTS

Let S and J be two bodies called Sun and Jupiter, of masses $m_S = 1 - \mu$ and $m_J = \mu$, $\mu \in (0, 1)$, respectively, describing circular orbits. The center of masses is placed at the origin. In a rotating frame (synodical coordinates) the equations of motion of a massless particle P under the gravitational action of S and J are

$$\begin{aligned} \ddot{x} - 2\dot{y} &= \Omega_x, \\ \ddot{y} + 2\dot{x} &= \Omega_y, \end{aligned} \tag{1.1}$$

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