# EJECTION AND COLLISION ORBITS OF THE SPATIAL RESTRICTED THREE-BODY PROBLEM

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(Received March, 1984; accepted July, 1984)

ABSTRACT. We begin by describing the global flow of the spatial two body rotating problem, $\mu = 0$. The remainder of the work is devoted to study the ejection and collision orbits when $\mu \geq 0$. We make use of the 'blow up' techniques to show that for any fixed value of the Jacobian constant the set of these orbits is diffeomorphic to $S^2 \times R$. Also we find some particular collision-ejection orbits.

## 1. INTRODUCTION

We consider the circular spatial restricted three-body problem (usually, the spatial restricted three-body problem) in a rotating coordinate system $q = (q_1, q_2, q_3)$ of rotational frequency equal to 1. In this frame (called synodical) we put the larger primary $m_1$ of mass $1 - \mu$ at the origin and the smaller primary $m_2$ of mass $\mu$ at the position $e_2 = (-1, 0, 0)$. The Hamiltonian which governs the motion of the zero mass particle $m_3$ is given by

$$H = \|p\|^2/2 + q_2 p_1 - q_1 p_2 - \|q\|^{-1} + \mu(\|q\|^{-1} - \|q - e_2\|^{-1} - p_2), \quad (1.1)$$

where $p = (p_1, p_2, p_3)$ are the momentum variables conjugate to $q$, and $\| \ \|$ is the Euclidean norm in $R^3$. It is clear that $C = -2H$ is a first integral of the Hamiltonian system associated with $H$. This integral is called the Jacobi integral. Note that our Jacobian constant differs from the usual one in the constant $\mu(1-\mu)$ (see [11]).

If we restrict the Hamiltonian (1.1) to $q_3 = p_3 = 0$, then we obtain the planar restricted three-body problem. The spatial restricted three-body problem is a one-parameter family of classical mechanical systems with three degrees of freedom of interest in Celestial Mechanics. When the parameter $\mu = 0$, we have the spatial two-body rotating problem. This system is integrable and its global flow is described in Section 2. For $\mu \in (0, 1)$ the system is not integrable, see [9] and [5].

*Celestial Mechanics* **35** (1985) 113–128. 0008-8714/85.15.
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