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Bifurcations of the Riccati Quadratic Polynomial Differential Systems

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In this paper, we characterize the global phase portrait of the Riccati quadratic polynomial differential system

$$\dot{x} = \alpha_2(x), \quad \dot{y} = ky^2 + \beta_1(x)y + \gamma_2(x),$$

with $(x, y) \in \mathbb{R}^2$, $\gamma_2(x)$ nonzero (otherwise the system is a Bernoulli differential system), $k \neq 0$ (otherwise the system is a Liénard differential system), $\beta_1(x)$ a polynomial of degree at most 1, $\alpha_2(x)$ and $\gamma_2(x)$ polynomials of degree at most 2, and the maximum of the degrees of $\alpha_2(x)$ and $ky^2 + \beta_1(x)y + \gamma_2(x)$ is 2. We give the complete description of the phase portraits in the Poincaré disk (i.e. in the compactification of $\mathbb{R}^2$ adding the circle $\mathbb{S}^1$ of the infinity) modulo topological equivalence.

Keywords: Bifurcation; topological equivalence; Riccati system; Poincaré compactification; dynamics at infinity.

1. Introduction and Statement of the Main Results

Numerous problems of applied mathematics are modeled by quadratic polynomial differential systems, see for instance [Maharaj *et al.*, 2016a; Mahomed *et al.*, 2020a]. Excluding linear systems, such systems are the ones with the lowest degree of complexity, and the large bibliography on the subject proves its relevance. We refer for example to the books [Ye *et al.*, 1984; Reyn, 2007; Artes *et al.*, 2021], and the surveys [Coppel, 1966; Chicone & Tian, 1982] are excellent introductory readings to the quadratic polynomial differential systems. Related problems can also be found in [Maharaj *et al.*, 2016b; Maharaj *et al.*, 2017; Mahomed *et al.*, 2020b].