



**Received:** 05 December, 2020

**Accepted:** 29 December, 2020

**Published:** 04 January, 2021

**\*Corresponding author:** Jaume Llibre, Department of Mathematics, Autonomous University of Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain, E-mail: [jllibre@mat.uab.cat](mailto:jllibre@mat.uab.cat)

**Keywords:** Central configuration; Circular restricted 4-body problem

<https://www.peertechz.com>



## Research Article

# Central configurations of the circular restricted 4-body problem with three equal primaries in the collinear central configuration of the 3-body problem

Jaume Llibre\*

Department of Mathematics, Autonomous University of Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

## Abstract

In this paper we classify the central configurations of the circular restricted 4-body problem with three primaries with equal masses at the collinear configuration of the 3-body problem and an infinitesimal mass.

## Introduction and results

The well-known Newtonian  $n$ -body problem concerns with the motion of  $n$  mass points with positive mass  $m_i$  moving under their mutual attraction in  $\mathbb{R}^d$  in accordance with Newton's law of gravitation.

The equations of the motion of the  $n$ -body problem are

$$\ddot{r}_i = - \sum_{j=1, j \neq i}^n \frac{m_j (r_i - r_j)}{r_{ij}^3}, \quad 1 \leq i \leq n,$$

where we have taken the unit of time in such a way that the Newtonian gravitational constant be one, and  $r_{i \in \mathbb{R}^d}$  ( $i=1, \dots, n$ ) denotes the position vector of the  $i$ -body,  $r_{ij} = |r_i - r_j|$  is the Euclidean distance between the  $i$ -body and the  $j$ -body.

The solutions of the 2-body problem (also called the Kepler problem) has been completely solved, but the solutions for the  $n$ -body for  $n > 2$ , is still an open problem.

For the Newtonian  $n$ -body problem the simplest possible motions are such that the configuration formed by the  $n$ -bodies is constant up to rotations and scaling, such motions are called the *homographic solutions* of the  $n$ -body problem, and are the unique known explicit solutions of the  $n$ -body problem when  $n > 2$ . Only some special configurations of particles are allowed in the homographic solutions of the  $n$ -body problem, called by Wintner [1] *central configurations*. Also, central configurations are of utmost importance when studying bifurcations of the hypersurfaces of constant energy and angular momentum, for more details see Meyer [2] and Smale [3]. These last years some central configurations have been used for different missions of the spacecrafts in the solar system, see for instance [4,5].

More precisely, let

$$M = m_1 + \dots + m_n, \quad c = \frac{m_1 r_1 + \dots + m_n r_n}{M},$$

be the *total mass* and the *center of masses* of the  $n$  bodies, respectively.