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A NOTE ON THE SET OF PERIODS FOR KLEIN BOTTLE MAPS

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By using the Nielsen fixed point theory we characterize the set of periods attained by continuous self-maps on the Klein bottle belonging to a given homotopy class.

1. Introduction. In dynamical systems, it is often the case that topological information can be used to study qualitative and quantitative properties (like the set of periods) of the system. This note deals with the problem of determining the set of periods (of the periodic points) of a continuous self-map of the Klein bottle. Our interest in this problem comes from the fact that the unique manifolds in dimensions 1 and 2 with zero Euler characteristic are the circle, torus and Klein bottle, and for the first two the structure of the set of periods of their continuous self-maps has been determined.

To fix terminology, suppose f is a continuous self-map on the manifold M. A *fixed point* of f is a point x in M such that $f(x) = x$. We shall call x a *periodic point of period* n if x is a fixed point of f^n but is not fixed by any f^k, for $1 \leq k < n$. We denote by $\mathrm{Per}(f)$ the set of natural numbers corresponding to periods of periodic orbits of f.

Even for continuous self-maps f on the circle the relation between the degree of f and the set $\mathrm{Per}(f)$ is interesting and nontrivial (see **[5]**, **[3]** and for more details **[2]**). Let $\mathbb{N}$ denote the set of natural numbers. Suppose f has degree d.

(1) For $d \notin \{-2, -1, 0, 1\}$, $\mathrm{Per}(f) = \mathbb{N}$.
(2) For $d = -2$, $\mathrm{Per}(f)$ is either $\mathbb{N}$ or $\mathbb{N} \setminus \{2\}$.
(3) For $d = -1, 0$, $\mathrm{Per}(f) \supset \{1\}$.
(4) For $d = 1$, the set $\mathrm{Per}(f)$ can be empty.

Recently, in **[1]** these results have been extended to continuous self-maps on the 2-dimensional torus, and many of them to the m-dimensional torus with $m > 2$.

The goal of this note is to provide a similar description of the set of periods for continuous self-maps on the Klein bottle, or simply *Klein bottle maps.*