ON THE NUMBER OF CENTRAL CONFIGURATIONS IN THE N-BODY PROBLEM

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(Received: 15 June, 1990; accepted: 18 December, 1991)

Abstract. Central configurations are critical points of the potential function of the n-body problem restricted to the topological sphere where the moment of inertia is equal to constant. For a given set of positive masses $m_1, \ldots, m_n$ we denote by $N(m_1, \ldots, m_n, k)$ the number of central configurations of the n-body problem in $\mathbf{R}^k$ modulus dilatations and rotations. If $N(m_1, \ldots, m_n, k)$ is finite, then we give a bound of $N(m_1, \ldots, m_n, k)$ which only depends of n and k.

Key words: N-body problem, central configuration

1. Introduction and Statement of the Results

A very old problem in Celestial Mechanics is the study of the central configurations of the n-body problem. Central configurations are initial positions of the bodies that lead to particular solutions of the n-body problem for which the ratios of the mutual distances between the bodies remain constant. There is an extensive literature concerning these solutions. For a classical background, see the sections on central configurations in (Wintner 1941) and (Hagihara 1970). For a modern background one can see (Smale 1970a, 1970b) and (Saari 1980). More recent work can be found in (Buck 1989, 1991; Cedó and Llibre 1989; Elmabsout 1988; Meyer 1987; Meyer and Schmidt 1988a, 1988b; Moeckel 1985, 1989; Palmore 1973, 1975a, 1975b; Pacella 1987; Perko and Walter 1985; Schmidt 1988; Shub 1970 and Simó 1977.

If $r_i = (x_i, y_i, z_i)$ is the position vector of the ith positive mass m_i relative to the center of mass of the system, then the particles form a *central configuration* at time t if and only if there exists some scalar λ such that $\ddot{r}_i = -\lambda r_i$ for $i = 1, 2, \ldots, n$. By replacing the acceleration vector $\ddot{r}_i$ by the force vector this equation becomes

$$\lambda r_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} m_j \frac{r_i - r_j}{r_{ij}^3} \quad \text{for} \quad i = 1, \ldots, n \,,$$

which is an equation which is independent of the dynamics. Here r_{ij} is the mutual distance between the ith and jth particles. It is well known that the constant λ in the above system is positive.