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# On the Extendable Regular Integrals of the $n$-Body Problem

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It is shown that the $n$-body problem in a $d$-dimensional space has no $C^1$-extendable regular integrals if $n \geqslant d+1$.

## 1. INTRODUCTION AND RESULTS

It is well known that the spatial $n$-body problem has ten classical integrals (center of mass, linear momentum, angular momentum and energy) and the elimination of nodes of Jacobi (some symmetry). Several attempts have been made to discover other integrals. Bruns (1887) proved that there cannot be any integral that is algebraic with respect to the coordinates and momenta, other than the classical integrals. Poincaré (1889) further proved that there cannot be any integral that is uniform with respect to the Keplerian elements, other than the classical integrals. Painlevé (1896–1898) extended Bruns' theorem to the case in which the integral contains algebraically the momentum only, while the coordinates are left arbitrary, for more details see [6]. We shall prove that the spatial $n$-body problem cannot be any $C^1$-integral of a special kind ($C^1$-extendable regular integral), other than the classical ones and the elimination of nodes of Jacobi, if $n \geqslant 4$ (see Theorem A and Corollary B, for more details).

We consider $n$ particles of positive masses $m_1,..., m_n$ moving in the space $R^d$ subject to Newton's gravitational law, i.e., the *n-body problem in d-dimensional space*. We are primarily interested in physical spaces where $d = 1, 2, 3$, but we will analyze all positive values of $d$. The *configuration space* of the $n$-body problem in $d$-dimensional space with the center of mass at the origin is the $(n-1)d$-dimensional manifold $M - \Delta$ where

$$M = \{x = (x_1,..., x_n) \in (R^d)^n : \sum m_i x_i = 0\},$$

$$\Delta = \bigcup \{\Delta_{ij} : 1 \leqslant i < j \leqslant n\}$$