A NOTE ON THE ENERGY LEVELS OF HAMILTONIAN SYSTEMS OF TWO DEGREES OF FREEDOM

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Abstract. *We use the "blow up" techniques and replace the zero velocity curve of a kind of Hamiltonian systems with two degrees of freedom by an invariant manifold, the zero velocity manifold. The Hamiltonian system extends smoothly over this manifold, and so we get a new flow on an augmented phase space. In this new phase space the energy levels always can be embedded into* R^3. *This makes easy the description of the flow.*

We are interested in Hamiltonian systems with two degrees of freedom

$$\frac{dq}{dt} = \frac{\partial H}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial H}{\partial q}, \tag{1}$$

where the *Hamiltonian or total energy* function

$$H(q,p) = \frac{1}{2} p^t M^{-1} p + V(q), \tag{2}$$

is such that M is the diagonal mass matrix 2x2 with positive entries m_1 and m_2, q is a point in Q (the configuration space) an open subset of R^2, the *potential energy* function $V: Q \longrightarrow R$ is sufficiently smooth and homogeneous of degree -k; i.e. $V(rq) = r^{-k}V(q)$, and p in R^2 is the momentum vector. This kind of Hamiltonian system are also studied in [4]. We shall restrict our attention to such potentials trough much that follows can be extended to other classes of potentials.

It is well known that H is a first integral of (1). So we can reduce the dimension of the system by one considering (1) as a vector field on the *energy level* $H^{-1}(e)$.