ON THE RESTRICTED THREE-BODY PROBLEM WHEN THE MASS PARAMETER IS SMALL*

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Abstract. We study some aspects of the restricted three-body problem when the mass parameter μ is sufficiently small. First, we describe the global flow of the two-body rotating problem, $\mu = 0$, and we use it for the analysis of the collision and parabolic orbits when $\mu \gtrsim 0$. Also we show that for any fixed value of the Jacobian constant and for any $\varepsilon > 0$, there exists a $\mu_0 > 0$ such that if the mass parameter $\mu \in [0, \mu_0]$, then the set of bounded orbits which are not contained in the closure of the set of symmetric periodic orbits has Lebesgue measure less than ε.

1. Introduction

We consider the circular planar restricted three-body problem (usually, the restricted three-body problem) in a rotating coordinate system $q = (q_1, q_2)$ of rotational frequency equal to 1. In this frame we put the larger primary m_1 of mass $1 - \mu$ at the origin and the smaller primary m_2 of mass μ at the position $e_2 = (-1, 0)$. The Hamiltonian which governs the motion of the zero mass particle m_3 is given by

$$H = \|p\|^2/2 + q_2 p_1 - q_1 p_2 - \|q\|^{-1} + \mu(\|q\|^{-1} - \|q - e_2\|^{-1} - p_2) \tag{1.1}$$

where $p = (p_1, p_2)$ are the momentum variables conjugate to the q. It is clear that $C = -2H$ is a first integral of the Hamiltonian system associated with H. This integral is called the Jacobi integral. Note that our Jacobian constant differs from the usual in the constant $\mu(1-\mu)$ (see [12]).

The goal of this paper is to study some aspects of the restricted three-body problem as the mass parameter μ is sufficiently small. First, in Section 2 we describe the global flow of the two-body rotating problem, $\mu = 0$, and use it, in Section 3, for the analysis of the collision and parabolic orbits when μ is small enough.

A solution of the restricted three-body problem has a collision with m_1 (resp. m_2) in the instant t_0 if the distance between m_3 and m_1 (resp. m_2) tends to zero as $t \to t_0$.

Our main results about the collision orbits are the following two theorems.

THEOREM A. *For the restricted three-body problem and for each value of the Jacobian constant, the set of orbits which end or begin at collision with m_1 or m_2 is topologically homeomorphic to a cylinder.*

THEOREM B. *For values of the mass parameter μ sufficiently small the following statements hold for the restricted three-body problem.*

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