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LIMIT CYCLES OF THE DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS ON THE CYLINDER*

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Dedicated to Professor Jibin Li on the occasion of his 80th birthday.

Abstract In order to understand the dynamics of the differential systems the limit cycles play a main role, but in general their study is not easy. These last years an increasing interest appeared for studying the limit cycles of some classes of discontinuous piecewise differential systems, due to the rich applications of this kind of differential systems.
Very few papers studied the limit cycles of the discontinuous piecewise differential systems in spaces different from the plane $\mathbb{R}^2$. Here we study the limit cycles of a class of discontinuous piecewise differential systems on the cylinder.

Keywords Limit cycles, discontinuous piecewise smooth system, differential systems on the cylinder.

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1. Introduction and statement of the main

Consider the following differential equation on the cylinder $(r, \theta) \in \mathbb{R} \times \mathbb{S}^1$

$$\frac{dr}{d\theta} = a_0(\theta) + a_1(\theta)r + a_2(\theta)r^2 + ... + a_n(\theta)r^n. \tag{1.1}$$

All the functions $a_i(\theta)$ are continuous and 2π-periodic in the variable θ. Equation (1.1) with $n = 1$ is a linear differential equation having at most one limit cycle, see for instance [4]. While for $n = 2$ it is a *Riccati* equation with at most two limit cycles, see [6]. For $n = 3$ it is an *Abel* equation. If $a_3(\theta) > 0$ Pliss [11] proved that the Abel equation has at most three limit cycles (see also [3, 8]). For $n \geq 4$ a constant sign in the leading coefficient a_n is not sufficient to bound uniformly the number of limit cycles (see [6, 8]). Lins Neto in [8] gave a example with at least $n + 3$ limit cycles for suitable functions a and f, for the Abel equation

$$\frac{dx}{d\theta} = \varepsilon f(\theta)x^3 + a(\theta)x^2 + \delta x,$$

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