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On the 16th Hilbert Problem for Discontinuous Piecewise Polynomial Hamiltonian Systems

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Abstract
In this paper we study the maximum number of limit cycles of the discontinuous piecewise differential systems with two zones separated by the straight line $y = 0$, in $y \geq 0$ there is a polynomial Hamiltonian system of degree m, and in $y \leq 0$ there is a polynomial Hamiltonian system of degree n. First for this class of discontinuous piecewise polynomial Hamiltonian systems, which are perturbation of a linear center, we provide a sharp upper bound for the maximum number of the limit cycles that can bifurcate from the periodic orbits of the linear center using the averaging theory up to any order. After for the general discontinuous piecewise polynomial Hamiltonian systems we also give an upper bound for their maximum number of limit cycles in function of m and n. Moreover, this upper bound is reached for some degrees of m and n.

Keywords Averaging method · Hilbert's 16th problem · limit cycles · discontinuous piecewise polynomial Hamiltonian systems

Mathematics Subject Classification 34C29 · 34C25 · 34C05

1 Introduction and Statement of Main Results

One of the most studied problems in the qualitatitve theory of the differential equations in the plane is to identify the maximum number of limit cycles that can exhibit a given class of differential systems. Thus a famous and challenging question is the Hilbert's 16th problem [22], which was proposed in 1900. In the second part of this question, Hilbert asked what is the maximum number of limit cycles that planar polynomial differential system of a given

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