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Limit cycles in piecewise polynomial systems allowing a non-regular switching boundary

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ABSTRACT

Continuing the investigation for the piecewise polynomial perturbations of the linear center $\dot{x} = -y, \dot{y} = x$ from Buzzi et al. (2018) for the case where the switching boundary is a straight line, in this paper we allow that the switching boundary is non-regular, i.e. we consider a switching boundary which separates the plane into two angular sectors with angles $\alpha \in (0, \pi]$ and $2\pi - \alpha$. Moreover, unlike the aforementioned work, we allow that the polynomial differential systems in the two sectors have different degrees. Depending on α and for arbitrary given degrees we provide an upper bound for the maximum number of limit cycles that bifurcate from the periodic orbits of the linear center using the averaging method up to any order. This upper bound is reached for the first two orders. On the other hand, we pay attention to the perturbation of the linear center inside this class of piecewise polynomial Liénard systems and give some better upper bounds in comparison with the one obtained in the general piecewise polynomial perturbations. Again our results imply that the non-regular switching boundary (i.e. when $\alpha \neq \pi$) of the piecewise polynomial perturbations usually leads to more limit cycles than the regular case (i.e. when $\alpha = \pi$) where the switching boundary is a straight line.

1. Introduction and statement of main results

In the qualitative theory of smooth differential systems, a classical and challenging objective is to determine the maximum number of the limit cycles bifurcating from the periodic orbits of the linear center $\dot{x} = -y, \dot{y} = -x$, when it is perturbed inside the family formed by all planar polynomial differential systems of the form

$$(\dot{x}, \dot{y}) = \left(-y + \sum_{i=1}^{N} \varepsilon^i f_i(x, y), x + \sum_{i=1}^{N} \varepsilon^i g_i(x, y)\right), \qquad (1)$$

where $|\varepsilon| > 0$ sufficiently small, and f_i and g_i are real polynomials of degree n. This can be regarded as a special case of the weak Hilbert's 16th problem, see [1–3]. For $|\varepsilon| > 0$ sufficiently small it was proved in [2] that $[N(n-1)/2]$ is an upper bound for the maximum number of the limit cycles of system (1) which can be obtained with the Melnikov method of order N, where as usual $[\cdot]$ denotes the integer part function. Since this upper bound obtained in [2] is not reached in general, up to now we still do not know what is the exact maximum number of limit cycles under the general polynomial perturbation (1) except some special families of perturbations, such as the Liénard family, i.e. $g_i(x, y) = 0$ and $f_i(x, y)$ is independent of the variable y, for which it was proved in [4] that at most $[(n-1)/2]$ limit cycles bifurcate and this number is reached due to [5].

As the discontinuity turns out to be ubiquitous in the real world (see for instance the papers in mechanical engineering [6,7], in neural sciences [8,9] and in electronic circuits [10,11], etc.), discontinuous piecewise smooth differential systems have attracted many researchers in recent years. Let Σ_α be the union of the non-negative x-axis and the ray starting at O and forming with the non-negative x-axis an angle $\alpha \in (0, \pi]$, Σ_α^+ and Σ_α^- be two angular sectors separated by Σ_α with angles α and $2\pi - \alpha$ respectively, see Fig. 1. It is worth mentioning that Σ_α is just the x-axis if $\alpha = \pi$. In this paper we consider the discontinuous piecewise polynomial differential systems of the form

$$(\dot{x}, \dot{y}) = \left(-y + \sum_{i=1}^{N} \varepsilon^i f_i^{\pm}(x, y), x + \sum_{i=1}^{N} \varepsilon^i g_i^{\pm}(x, y)\right) \quad \text{if} \quad (x, y) \in \Sigma_\alpha^{\pm}, \qquad (2)$$

where f_i^+ and g_i^+ (respectively f_i^- and g_i^-) are real polynomials of degree n (respectively m). In this case we say that the *degree* of system (2) is (n, m). Throughout this paper we will restrict our attention to the case of $n \geq m \geq 1$.

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