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ORIGINAL ARTICLE

Limit cycles bifurcating from a zero-Hopf equilibrium of a 3-dimensional continuous differential system

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Abstract
A zero-Hopf equilibrium of a differential system in $\mathbb{R}^3$ is an equilibrium point whose linear part has eigenvalues 0 and $\pm\omega i$ with $\omega > 0$. We provide necessary and sufficient conditions for the existence of two or one limit cycles bifurcating from a zero-Hopf equilibrium of the following 3-dimensional Lypschizian differential systems

$$\begin{aligned}\dot{x} &= y,\\ \dot{y} &= z,\\ \dot{z} &= -a|x| - y + 3y^2 - xz - b,\end{aligned}$$

when $a = b = 0$. Note that due to the existence of an absolute value the vector field associated to this system is only Lypschitz. We shall prove that these limit cycles persist for $ab > 0$ with a and b sufficiently small, i.e. when the differential system has no equilibria. Additionally we provide an estimation of the size of the bifurcating small limit cycles and also characterize their kind of stability or instability. We remark that there are no works which study the zero-Hopf bifurcation of nonsmooth differential systems as we do in this paper, and that the algorithm that we use here can be applied for studying the zero-Hopf bifurcation of an arbitrary nonsmooth differential system.

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