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Research paper

Flow map parameterization methods for invariant tori in Hamiltonian systems

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ABSTRACT

The goal of this paper is to present a methodology for the computation of invariant tori in Hamiltonian systems combining flow map methods, parameterization methods, and symplectic geometry. While flow map methods reduce the dimension of the tori to be computed by one (avoiding Poincaré maps), parameterization methods reduce the cost of a single step of the derived Newton-like method to be proportional to the cost of a FFT. Symplectic properties lead to some magic cancellations that make the methods work. The multiple shooting version of the methods are applied to the computation of invariant tori and their invariant bundles around librational equilibrium points of the Restricted Three Body Problem. The invariant bundles are the first order approximations of the corresponding invariant manifolds, commonly known as the whiskers, which are very important in the dynamical organization and have important applications in space mission design.

1. Introduction

Hamiltonian systems are frequently found in physical and engineering applications, where challenging problems continuously emerge, of both theoretical and practical nature. The developement of efficient methods for computing invariant tori carrying quasi-periodic motion is a driving force in the applications of Hamiltonian systems (see e.g. [52] for early references), in areas such as plasma physics, semiclassical quantum theory, accelerator theory, magnetohydrodynamics, oceanography and, of course, celestial mechanics. While Lagrangian (maximal dimensional) invariant tori are important in stability studies, partially hyperbolic invariant tori are also important in studies of diffusion and chaos in Hamiltonian systems. An important problem in astrodynamics is the design of station keeping orbits lying on partially hyperbolic invariant tori around collinear libration points in RTBP approximations, for which the stable manifolds are sort of entry lanes, and the unstable manifolds are the exit lanes (see [21] for a survey of early libration points missions, and the web pages of space agencies for many newer ones). This will be the guiding problem of this paper to fix a framework.

To date, one of the most succesful approaches to compute invariant tori falls in the category of *numerical Fourier methods*, in which parameterizations of tori are given by (truncated) Fourier expansions, and the arising discretized invariance equations (using e.g. collocation) are solved by numerical methods such as Newton's method (see e.g. [1,14,15,35,47] for several variants of this approach in differents contexts). In spite of the relatively simple formulation of the approach, the main

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