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$\mathcal{C}^1$ SELF–MAPS ON SOME COMPACT MANIFOLDS WITH INFINITELY MANY HYPERBOLIC PERIODIC ORBITS

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Abstract. The aim of the present work is to provide sufficient conditions for having infinitely many periodic points for $\mathcal{C}^1$ self–maps having all their periodic orbits hyperbolic and defined on a compact manifold without boundary. The tool used for proving our results is the Lefschetz fixed point theory.

1. Introduction and statement of the main results

Let $\mathbb{M}$ be a topological space and $f : \mathbb{M} \to \mathbb{M}$ be a continuous map. A point x is called *fixed* if $f(x) = x$ and *periodic* of period k if $f^k(x) = x$ and $f^i(x) \neq x$ if $1 \leq i < k$. By $\mathrm{Per}(f)$ we denote the set of periods of all the periodic points of f.

If $x \in \mathbb{M}$ the set $\{x, f(x), f^2(x), \ldots, f^n(x), \ldots\}$ is called the *orbit* of the point x. Here f^n means the composition of n times of f with itself. To study the dynamics of the map f is to study all the different kind of orbits of f. Of course, if x is a periodic point of f of period k, then its orbit is $\{x, f(x), f^2(x), \ldots, f^{k-1}(x)\}$, and it is called a *periodic orbit*.

In this paper we study the periodic structure of $\mathcal{C}^1$ self–maps f defined on a given compact manifold $\mathbb{M}$ without boundary. Often the periodic orbits play an important role in the general dynamics of a map, for studying them we can use topological information. Perhaps the best known example in this direction are the results contained in the seminal paper entitle *Period three implies chaos* for continuous self–maps on the interval, see [12].

For continuous self–maps on compact manifolds one of the most useful tools for proving the existence of fixed points and in general of periodic points, is the

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