ON THE PERIODIC ORBITS OF THE CONTOPOULOS HAMILTONIAN

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1. INTRODUCTION

Looking for a third integral in the motion of a particle in a galactic potential with cylindrical symmetry, Contopoulos[3,4] was lead to the Hamiltonian

$$H(x,y) = \frac{1}{2}(y_1^2 + y_2^2) + \frac{1}{2}(x_1^2 + x_2^2) - x_1 x_2^2 ,$$

where the position is given by $x = (x_1, x_2)$ and the momentum by $y = (y_1, y_2)$. The Hamiltonian system X_H is

$$\dot{x}_1 = y_1 , \quad \dot{x}_2 = y_2 , \quad \dot{y}_1 = -x_1 + x_2^2 , \quad \dot{y}_2 = -x_2 + 2x_1 x_2 .$$

This system has the origin $Q_1 = (0,0,0,0)$ as a critical point at energy level $h = 0$ with repeated eigenvalues $\pm i$, and two additional critical points $Q_2 = (2^{-1}, -2^{-1/2}, 0, 0)$ and $Q_3 = (2^{-1}, 2^{-1/2}, 0, 0)$ at energy level $h = 1/8$ with eigenvalues ± 1 and $\pm 2^{1/2} i$.

This communication surveys and states results on the simple periodic orbits of X_H which start in their critical points.

2. THE FAMILIES OF SIMPLE PERIODIC ORBITS

We restrict to some energy level $H = h$. Cutting it by $x_2 = 0$ we get a 2-dimensional manifold S. Then consider the Poincaré map T which maps S into itself in the following way. Take a point $p \in S$ with coordinates (x_1, y_1), $x_2 = 0$ and $y_2 \geq 0$ obtained from $H(p) = h$. Then T(p) is the point given by the next cut of S by the orbit through p, if it exists. Fixed points under T are associated with the so called simple periodic orbits of X_H.

Table I summarizes the results about the existence of the simple periodic orbits of X_H which start in the critical points Q_i, for $i = 1,2,3$.

In a similar way to [6] we have proved that Figure 1 (resp. Figure 2) gives us the projections on the position plane of the periodic orbits π_i for $i = 1,2,3,4,5,6$ (resp. $i = 1,3,5,6,7,8$) when the energy $h \in (0, 1/8)$ (resp. $h \in (1/8, +\infty)$).

<u>Proposition</u>: For all h the orbits π_i for $i = 1,2,3,4,5,6$ are mutually linked in the energy level h, if they exist.