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Local cyclicity in low degree planar piecewise polynomial vector fields

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In this work, we are interested in isolated crossing periodic orbits in planar piecewise polynomial vector fields defined in two zones separated by a straight line. In particular, in the number of limit cycles of small amplitude. They are all nested and surrounding one equilibrium point or a sliding segment. We provide lower bounds for the local cyclicity for planar piecewise polynomial systems, $M_p^c(n)$, with degrees 2, 3, 4, and 5. More concretely, $M_p^c(2) \geq 13$, $M_p^c(3) \geq 26$, $M_p^c(4) \geq 40$, and $M_p^c(5) \geq 58$. The computations use parallelization algorithms.

1. Introduction

The study of piecewise or nonsmooth differential systems was started by the school of Andronov, see for example [1]. Many problems of engineering can be modeled by this class of systems, see [2]. Recently, they also appear modeling different situations in physics and biology, see [3]. One of the most studied situations in the plane is given by two vector fields defined in two half-planes separated by a straight line. As in the case of the classical qualitative theory of polynomial systems, the study of the number and location of the isolated periodic orbits, also called limit cycles, have received special attention. See for example [4–9]. In particular, it can be seen as an extension of the 16th-Hilbert problem for planar piecewise polynomial vector fields. Ilyashenko, in [10], presented an updated summary of the status of this problem, proposed by Hilbert more than one hundred years ago.

In this work, our main interest is the study of the number of limit cycles bifurcating from the origin in the class of piecewise polynomial differential equations defined in two zones separated by a straight line. In

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