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Lower bounds for the local cyclicity of centers using high order developments and parallelization

Luiz F.S. Gouveia [a,b], Joan Torregrosa [a,*]

[a] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*
[b] *Departamento de Matemática, Universidade Estadual Paulista, 15054-000 São José do Rio Preto, Brazil*

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Abstract

We are interested in small-amplitude isolated periodic orbits, so-called limit cycles, surrounding only one equilibrium point, that we locate at the origin. We develop a parallelization technique to study higher order developments, with respect to the parameters, of the return map near the origin. This technique is useful to study lower bounds for the local cyclicity of centers. We denote by $M(n)$ the maximum number of limit cycles bifurcating from the origin via a degenerate Hopf bifurcation for a polynomial vector field of degree n. We get lower bounds for the local cyclicity of some known cubic centers and we prove that $M(4) \geq 20$, $M(5) \geq 33$, $M(7) \geq 61$, $M(8) \geq 76$, and $M(9) \geq 88$.

MSC: primary 34C23, 34C25, 34C07

Keywords: Small-amplitude limit cycle; Polynomial vector field; Center cyclicity; Lyapunov constants; Higher-order developments and parallelization

1. Introduction

Hilbert early last century presented a list of problems that almost all of them are solved. One problem that still is open is the second part of the 16th Hilbert's problem: It consists in determine

* Corresponding author.
E-mail addresses: fernandosg@mat.uab.cat, fernando.gouveia@unesp.br (L.F.S. Gouveia), torre@mat.uab.cat (J. Torregrosa).

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