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Phase portraits of the quadratic polynomial Liénard differential systems

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We classify the global phase portraits in the Poincaré disc of the quadratic polynomial Liénard differential systems

$$\dot{x} = y, \quad \dot{y} = (ax + b)y + cx^2 + dx + e,$$

where $(x, y) \in \mathbb{R}^2$ are the variables and a, b, c, d, e are real parameters.

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1. Introduction

A *quadratic polynomial differential system* is a system of the form

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y),$$

where P and Q are polynomials in the variables x and y, and the maximum of the degrees of P and Q is two.

The quadratic polynomial differential systems and their applications have been studied intensively these last 30 years, see for instance the exhaustive bibliography about these systems in the books of Reyn [40] and Ye Yanqian [48]. More concretely, classes of quadratic systems that have been studied are: homogeneous (see [13, 34, 36]), bounded (see [11, 16, 19]), having a star nodal point (see [6]), chordal (see [22, 23]), with a weak focus of second or third order (see [4, 5, 29, 32]), with four infinite critical points and one invariant straight line (see [42]), Hamiltonian (see [2]), gradient (see [10]), having a focus and one antisaddle (see [3]), integrable