STATION KEEPING OF A QUASIPERIODIC HALO ORBIT USING INVARIANT MANIFOLDS

C Simó G Gómez J Llibre & R Martínez

Dept. de Matemàtica Aplicada i Anàlisi, Facultat de Matemàtiques, Univ. de Barcelona.

Dept. de Matemàtiques, E.T.S.E.I.B. Univ. Politècnica de Catalunya.

Dept. de Matemàtiques, Facultat de Ciències, Univ. Autònoma de Barcelona.

ABSTRACT

We study the station keeping of quasiperiodic orbits near a periodic halo orbit of the Earth+Moon-Sun system. The full solar system is considered as a perturbation of the RTBP. For the RTBP the halo orbits near the libration points L_1 ot L_2 are well known but they are unstable. In the full solar system the periodic orbits are lost but the system still has quasiperiodic solutions relatively close to the previous halo orbits. They are approximately obtained analytically and refined numerically. These orbits can be taken as nominal orbits for halo missions. They inherit the unstable character of the halo orbits. Despite they are no longer periodic we can still introduce their invariant manifolds, which are approximated by analytical and numerical solutions of the variational equations. Simple geometrical considerations allow us to develop a very cheap station keeping algorithm.

Keywords: Halo orbits, quasiperiodic orbits, instability, invariant manifolds, station keeping.

1. INTRODUCTION AND GENERAL OVERVIEW

As it is well known since Euler and Lagrange epoch, the three body problem and, in particular, the restricted three body problem,RTBP, has 5 libration points. The RTBP describes the motion of a particle, of very small mass (the spacecraft), under the gravitational attraction of two massive bodies (Sun and Earth+Moon barycenter), that are in circular motion around their center of masses (Ref. 14). Synodical coordinates are introduced to keep fixed both main bodies. This is accomplished by introducing axes which rotate with the same angular velocity as that of the main bodies.

The libration points are equilibrium points in these coordinates. At them the attraction of the massive bodies is exactly cancelled by the centrifugal force. Three of them are in the line joining both primaries and the one between primaries is called L_1. The two remaining libration points are the equilateral ones. They will not be discussed here.

The previous situation is quite idealistic. However, real world can be considered as a not too large perturbation of this ideal behaviour in many cases. We will focus on the Sun-Barycenter system for which we modify slightly the mass of the Sun to satisfy Kepler's third law, and we add the following terms:
(a) Due to the real motion of the Barycenter and to the previously skipped part of the mass of the Sun,
(b) Due to the fact that instead of the Barycenter there is the Earth-Moon system,
(c) The effect of planets, mainly Venus and Jupiter,
(d) The solar radiation pressure.
In this field of forces the dynamical libration points do not subsist. However we can introduce geometrical libration points, with respect to Sun and Barycenter, given by the same relations that are obtained in the RTBP. A particle placed on them moves slowly, provided it is not too far from them.

Due to these nice properties of the libration points, they are suitable for space missions. The L_1 point of the Sun-Barycenter system is an ideal site for a solar observatory. The Sun's surface is always available, the Earth is far enough to have low noise and near enough to allow for good communications. However, L_1 is not suitable because the signals from the spacecraft disappear in the solar noise. Some angular deviation from the Sun is required. There are periodic orbits (the so called halo orbits) in the RTBP which do exactly what we want: they are not too far from L_1 and the angular distance to the Sun is big enough. For previous work on these orbits see Ref. 3, 4, 5, 7. In this work we deal with this type of orbits and we study how they are modified when the perturbations (a) to (d) are considered. The definition of the nominal orbit can be strict or not, that is, we can force the spacecraft to follow the nominal orbit closely, or we can only ask to the spacecraft to be not too far from a given path. For the ISEE-3 mission the second procedure was adopted (Ref. 7). In this work we propose to follow the orbit closely. This means a substantial reduction in the expected fuel consumption for station keeping. In the real world periodic orbits no longer subsist. They are replaced by quasiperiodic ones (Ref. 1). A quasiperiodic motion can be seen as superposition of harmonics with different incommensurable frequencies. A halo orbit should be replaced by a nearby quasiperiodic orbit, and the possible escaping components should be avoided because the halo orbits of small and medium size are unstable. A particle starting at a distance d from a halo orbit, leaves this orbit as $d\exp(mt)$. For the L_1 case considered, and t in days, a typical value of m is 0.042. In one year an initial error will be multiplied by more than $3\cdot 10^6$. An important fact is that, in the range of interest only one unstable direction appears. For the basic definitions and