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A new sufficient condition in order that the real Jacobian conjecture in $\mathbb{R}^2$ holds

Jaume Giné [a,*], Jaume Llibre [b]

[a] *Departamento de Matematica, Universitat de Lleida, Avda. Jaume II, 69, 25001 Lleida, Catalonia, Spain*

[b] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*

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Abstract

Let $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that $\det(DF(x, y))$ is nowhere zero and $F(0, 0) = (0, 0)$. In this work we give a new sufficient condition for the injectivity of F. We also state a conjecture when $\det(DF(x, y)) = \text{constant} \neq 0$ and $F(0, 0) = (0, 0)$ equivalent to the Jacobian conjecture.

MSC: primary 14R15; secondary 35F05, 35A30

Keywords: Real Jacobian conjecture; Global injectivity; Center

1. Introduction and statement of the main result

Let $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a smooth map such that the determinant of the Jacobian matrix $\det(DF)$ is nowhere zero. By the Inverse Theorem such a map F is a local diffeomorphism. However this map is not always an injective map. But with some additional conditions it holds that F is a global diffeomorphism, see for instance [11,16,24].

The *Jacobian conjecture*, stated by Keller [22] in 1939, states that when F is a polynomial map and $\det(DF(x, y)) = \text{constant} \neq 0$, then F is injective. Many authors have work in this conjecture, see for instance the surveys [2], and [15] on the Jacobian conjecture and related problems, but for the moment this conjecture remains open.

* Corresponding author.

E-mail addresses: jaume.gine@udl.cat (J. Giné), jllibre@mat.uab.cat (J. Llibre).

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