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Formal Weierstrass Nonintegrability Criterion for Some Classes of Polynomial Differential Systems in $\mathbb{C}^2$

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In this paper, we present a criterion for determining the formal Weierstrass nonintegrability of some polynomial differential systems in the plane $\mathbb{C}^2$. The criterion uses solutions of the form $y = f(x)$ of the differential system in the plane and their associated cofactors, where $f(x)$ is a formal power series. In particular, the criterion provides the necessary conditions in order that some polynomial differential systems in $\mathbb{C}^2$ would be formal Weierstrass integrable. Inside this class there exist non-Liouvillian integrable systems. Finally we extend the theory of formal Weierstrass integrability to Puiseux Weierstrass integrability.

Keywords: Weierstrass integrability; Liouville integrability; polynomial differential system.

1. Introduction

To determine when a differential system in $\mathbb{C}^2$ has or has not a first integral is one of the main problems in the qualitative theory of differential systems. The Liouville integrability is one of the most important theories of integrability starting with Darboux in [Darboux, 1878a, 1878b] and strongly developed during the last decades [Dumortier *et al.*, 2006; Giné, 2007; Singer, 1992]. This theory is based on the existence of invariant algebraic curves and their multiplicity through the exponential factors. Recently generalizations on the Liouville integrability theory have been done in several works, see [Giné & Grau, 2010; Giné *et al.*, 2013; Llibre & Zhang, 2009, 2010; Zhang, 2017].

Of course, there exist differential systems which are integrable, i.e. with an explicit first integral, and that are non-Liouvillian integrable, an example is given below. Hence a natural question is: How to detect these non-Liouvillian integrable systems? In this work, we shall see that this detection is possible for some class of polynomial differential systems through a new criterion that detects formal Weierstrass nonintegrability. We also apply the criterion to some polynomial differential systems. But we must start providing some preliminary definitions and results.

Consider the polynomial differential system in the plane $\mathbb{C}^2$

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where the functions P and Q are polynomials in the complex variables x and y, i.e. $P, Q \in \mathbb{C}[x, y]$. The *degree* of system (1) is $m = \max\{\deg P, \deg Q\}$.