Criteria on the existence of limit cycles in planar polynomial differential systems

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Abstract

We summarize known criteria for the non-existence, existence and on the number of limit cycles of autonomous real planar polynomial differential systems, and also provide new results. We give examples of systems which realize the maximum number of limit cycles provided by each criterion. In particular we consider the class of differential systems of the form $\dot{x} = P_n(x, y) + P_m(x, y)$, $\dot{y} = Q_n(x, y) + Q_m(x, y)$, where n, m are natural numbers with $m > n \geq 1$ and (P_i, Q_i) for $i = n, m$, are quasi-homogeneous vector fields.

1. Introduction and statement of the main results

Poincaré in [34] defined the concept of limit cycle of a planar differential system and started to study it intensively. Later on the limit cycles were studied by van der Pol [39], Liénard [24], Andronov [3], ..., and they gave account of how difficult is their control. In fact one of the main problems in the qualitative theory of real planar differential systems