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Limit Cycles of Piecewise-Continuous Differential Systems Formed by Linear and Quadratic Isochronous Centers II

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We study the crossing periodic orbits and limit cycles of the planar piecewise-continuous differential systems separated by the straight-line $x = 0$ having in $x > 0$ the general quadratic isochronous center $\dot{x} = -y + x^2$, $\dot{y} = x(1 + y)$ after an affine transformation, and in $x < 0$ an arbitrary quadratic isochronous center except for the quadratic isochronous center $\dot{x} = -y + x^2 - y^2$, $\dot{y} = x(1 + 2y)$ which has been studied in [Ghermoul *et al.*, 2021]. For these piecewise-continuous differential systems the upper bound of crossing limit cycles is 2, and there are specific examples having one crossing limit cycle.

Keywords: Limit cycle; isochronous quadratic center; continuous piecewise linear differential system; first integral.

1. Introduction

In the qualitative theory of planar differential systems a *limit cycle* is an isolated periodic solution in the set of all periodic solutions, which has remained the most sought after solution when modeling physical systems in the plane. As far as we know the notion of limit cycle appeared in the year 1885 in the work of Poincaré [1928].

Most of the early examples in the theory of limit cycles in planar differential systems were commonly related to practical problems with mechanical and electronic systems, but periodic behavior appears in all branches of the sciences. To determine the existence or nonexistence of limit cycles is one of the more difficult goals in the qualitative theory of planar differential equations. A large amount of references deals with the subject of limit cycles, many of them motivated for the famous Hilbert's 16th problem, see for details [Hilbert, 1900; Ilyashenko, 2002; Li, 2003].

Since the 1930's, the study of limit cycles also became important in the continuous and discontinuous piecewise differential systems separated by a straight line, due to their applications to mechanics, electrical circuits, ... see for instance, the books [Andronov *et al.*, 1966; di Bernardo *et al.*, 2008; Simpson, 2010] and the references therein.