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Discrete Melnikov functions

Armengol Gasull[a,b] and Claudia Valls[c]

[a]Departament de Matemàtiques, Universitat Autònoma de Barcelona, Barcelona, Spain; [b]Centre de Recerca Matemàtica, Campus de Bellaterra, Barcelona, Spain; [c]Dep. de Matemática, Instituto Superior Técnico, Lisboa, Portugal

ABSTRACT
We consider non-autonomous N-periodic discrete dynamical systems of the form $r_{n+1} = F_n(r_n, \varepsilon)$, having when $\varepsilon = 0$ an open continuum of initial conditions such that the corresponding sequences are N-periodic. From the study of some variational equations of low order, we obtain successive maps, that we call discrete Melnikov functions, such that the simple zeroes of the first one that is not identically zero control the initial conditions that persist as N-periodic sequences of the perturbed discrete dynamical system. We apply these results to several examples, including some Abel-type discrete dynamical systems and some non-autonomous perturbed globally periodic difference equations.

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1. Introduction and statement of the main results

The interest in the study of non-autonomous periodic discrete dynamical systems has been increasing in the last years, among other reasons, because they are good models for describing the dynamics of biological and ecological systems that vary periodically, either due to external disturbances or for effects of seasonality, see, for instance, [2,11–13,15–17] and the references therein.

Consider non-autonomous discrete dynamical systems of the form

$$r_{n+1} = f_n(r_n), \quad r_n \in \mathbb{R}^d,\ n \in \mathbb{N}, \tag{1}$$

where $d \in \mathbb{N}^+$, and f_n is an N-periodic sequence of real smooth invertible maps such that $f_m = f_{m+N}$ for all $m \in \mathbb{N}$. Here, $f_n : \mathcal{U} \subset \mathbb{R}^d \to \mathcal{U}$ being $\mathcal{U}$ an open set of $\mathbb{R}^d$. Given an initial condition $r_0 = \rho \in \mathcal{U}$ we will denote by $r_n = \varphi_n(\rho)$ the sequence defined by (1). For convenience, for $n > 0$, we write $f_{n,n-1,\dots,1,0} = f_n \circ f_{n-1} \cdots \circ f_1 \circ f_0$. Then, for $n > 0$,

$$r_n = \varphi_n(\rho) = f_{n-1,n-2,\dots,1,0}(\rho). \tag{2}$$

It is well-known that given an N-periodic discrete dynamical system (1), it can be understood via the so called *composition map* $f_{N-1,N-2,\dots,1,0}$. For instance, if all maps share a

CONTACT Armengol Gasull gasull@mat.uab.cat Departament de Matemàtiques, Universitat Autònoma de Barcelona, Edifici Cc, Barcelona, 08193 Cerdanyola del Vallès, Spain; Centre de Recerca Matemàtica, Campus de Bellaterra, Edifici Cc, Barcelona, 08193 Cerdanyola del Vallès, Spain