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PIECEWISE LINEAR DIFFERENTIAL SYSTEMS WITH AN ALGEBRAIC LINE OF SEPARATION

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Abstract. We study the number of limit cycles of planar piecewise linear differential systems separated by a branch of an algebraic curve. We show that for each $n \in \mathbb{N}$ there exist piecewise linear differential systems separated by an algebraic curve of degree n having $[n/2]$ hyperbolic limit cycles. Moreover, when $n = 2, 3$, we study in more detail the problem, considering a perturbation of a center and constructing examples with 4 and 5 limit cycles, respectively. These results follow by proving that the set of functions generating the first order averaged function associated to the problem is an extended complete Chebyshev system in a suitable interval.

1. Introduction and statement of main results

For planar polynomial differential systems there still stands the unsolved Hilbert's 16th problem, whose second part asks for the maximum number of limit cycles and their distribution in terms of their degree, see for instance [9, 11]. As we know, planar linear differential systems have no limit cycle, whereas the situation is different for piecewise linear differential systems. For them, in the case when they are defined in two zones separated by a straight line, it is known that there are examples with 3 limit cycles, see [4, 7, 8, 13]. It is yet an open problem to know the maximum number of limit cycles that this type of systems can have.

We address to a very related question. Let $\mathcal{PL}_n$ be the set of planar piecewise linear differential systems with two zones separated by a branch of an algebraic curve of degree n, C_n, where in each of the two zones the differential system is linear. As usual, limit cycles will be periodic solutions isolated in the set of all periodic solutions. Here, the definition of solution in the two zones scenario is the one given in [5, 6, 15]. We remark that in this setting there are crossing and sliding limit cycles. In this paper, when we refer to limit cycles we will mean the ones of crossing type, even when we do not explicitly mention it. Recall that this type of periodic solutions cut the discontinuity curve only at finitely many points, both vector fields are transversal to this curve at each of these points and, moreover, both vectors fields point towards the same component of $\mathbb{R}^2 \setminus C_n$ at each of them.

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