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PERIODIC ORBITS OF DISCRETE AND CONTINUOUS DYNAMICAL SYSTEMS VIA POINCARÉ-MIRANDA THEOREM

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Abstract. We present a systematic methodology to determine and locate analytically isolated periodic points of discrete and continuous dynamical systems with algebraic nature. We apply this method to a wide range of examples, including a one-parameter family of counterexamples to the discrete Markus-Yamabe conjecture (La Salle conjecture); the study of the low periods of a Lotka-Volterra-type map; the existence of three limit cycles for a piecewise linear planar vector field; a new counterexample of Kouchnirenko conjecture; and an alternative proof of the existence of a class of symmetric central configuration of the $(1+4)$-body problem.

1. **Introduction and main results.** Periodic orbits are one of the main objects of study of the theory of dynamic systems. A priori there are many ways to prove the existence of periodic orbits, for instance one can try to apply the plenty of available fixed point theorems [15] or results guaranteeing the existence of zeros, since periodic orbits are always solutions of equations of the form $G(\mathbf{x}) = \mathbf{x}$, where G is a return map in the continuous case, and $G = F^p$ for some $p \in \mathbb{N}$ in the case of a discrete system given by a map F. However when one tries to apply these results to a particular case it is not always easy to find effective ways to check the hypotheses. An example of this fact appears when trying to use the Newton-Kantorovich Theorem [19]. By using this approach, some bounds of the partial derivatives of the involved functions must be obtained. The work done in [3] exemplifies clearly the difficulties of this approach.

In this paper we present an effective procedure to prove the existence, determine the number and locate periodic orbits of dynamical systems of both discrete and continuous nature. This procedure is explained in detail in the next sections. As we will see, one of the main features of this procedure is the use of the Poincaré-Miranda

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Key words and phrases. Poincaré-Miranda theorem, periodic orbits, Lotka-Volterra maps, Thue-Morse maps, discrete Markus-Yamabe conjecture, Kouchnirenko conjecture, Limit cycles, planar piecewise linear systems, central configurations.

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