JOURNAL OF DIFFERENTIAL EQUATIONS 91, 327–335 (1991)

Global Asymptotic Stability of Differential Equations in the Plane

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Received August 23, 1989

1. INTRODUCTION

The problem of determining the basin of attraction of equilibrium points is of paramount importance for applications of stability theory.

Local conditions which guarantee the existence of small basins of attraction, such as $\operatorname{tr} L<0$ and $\det L>0$, where L is the linear part of the planar system at an equilibrium point, are well known.

This paper is concerned with sufficient conditions which guarantee that the basin of attraction of an equilibrium point of a $\mathscr{C}^1$ planar system of differential equations $x'=f(x)$ is the whole x-space $\mathbb{R}^2$.

In this context, the fundamental problem, yet unsolved, is the following: Consider an autonomous system of differential equations

$$x'=f(x) \qquad ('=d/dt), \tag{S}$$

where $x=(x_1, x_2)$ and $f(x)=(f_1(x_1, x_2), f_2(x_1, x_2))$.

Let $\mathscr{F}$ be the class of $\mathscr{C}^1$ maps $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ such that

(i) the origin $0=(0,0)$ is a critical point of (S), i.e., $f(0)=0$,

(ii) $\operatorname{tr} Df(x)<0$ on $\mathbb{R}^2$,

(iii) $\det Df(x)>0$ on $\mathbb{R}^2$,

where $Df(x)=(\partial f_i/\partial x_j)$ is the Jacobian matrix.

* Partially supported by FAPERJ Grant E29/170.64/89.

† Partially supported by CIGYT Grant PB86-0351.

‡ Partially supported by the Centre de Recerca Matemàtica de l'Institut d'Estudis Catalans.

0022-0396/91 $3.00