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ON THE NONSINGULAR QUADRATIC DIFFERENTIAL EQUATIONS IN THE PLANE

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ABSTRACT. We point out that the maximum number of inseparable leaves of nonsingular polynomial differential equations of degree two is 3, and we present an example with exactly 3 inseparable leaves.

A nonsingular differential equation in two real variables defines a foliation of the plane. It is well known that the topological classification of such foliations depends only on the number of inseparable leaves and the way they are distributed in the plane (see [**K1**, **K2** or **HR**]). Two leaves (or trajectories) L_1 and L_2 are said to be *inseparable* if for any arcs T_1 and T_2 respectively transversal to L_1 and L_2 there are infinitely many leaves which intersect both T_1 and T_2.

For polynomial foliations of degree n, i.e. defined by a polynomial vector field (P, Q) where $\max\{\deg(P), \deg(Q)\} = n$, it is known that the number of inseparable leaves is at most $2n$ (see [**M** or **SS**]). A construction leading to examples with $2n-4$ inseparable leaves for all $n \geq 4$ can be found in [**P**]. In [**CP2**] it is studied the case $n = 3$. In [**CP1**] it is claimed that the case $n = 2$ has at most 2 inseparable leaves. This claim is not true because the easily integrable quadratic system $\dot{x} = 1 + xy$, $\dot{y} = y^2/m$ with $m < -1$ studied in [**GLL**] has 3 inseparable leaves given by $y = 0$ and the two branches of the hyperbola $xy = -m/(m+1)$.

The main result of [**GLL**] shows that for nonsingular quadratic differential equations there are 23 different topological phase portraits in the Poincaré sphere. As a consequence there follows the next theorem.

THEOREM. *Consider the foliation defined by the differential equation* $Pdx + Qdy = 0$, *where* P *and* Q *are polynomials of degree at most* 2 *in the two real variables and such that* $P^2 + Q^2$ *never vanishes. Then there is a homeomorphism taking leaves of* $Pdx + Qdy = 0$ *to leaves of one of the three following foliations*: $dx = 0$, $xdx + (1 - x^2)dy = 0$, $(y^2/2)dx + (1 + xy)dy = 0$.

From this result we obtain immediately the following corollary, which answers a question raised in [**CT**] about the number of Reeb components that a quadratic system in the plane may admit.

COROLLARY. *There are at most two Reeb components for a nonsingular quadratic vector field in the plane.*

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