

Parrondo's paradox for homoeomorphisms

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We construct two planar homoeomorphisms f and g for which the origin is a globally asymptotically stable fixed point whereas for $f \circ g$ and $g \circ f$ the origin is a global repeller. Furthermore, the origin remains a global repeller for the iterated function system generated by f and g where each of the maps appears with a certain probability. This planar construction is also extended to any dimension >2 and proves for first time the appearance of the dynamical Parrondo's paradox in odd dimensions.

Keywords: Dynamical Parrondo's paradox; fixed points; local and global asymptotic stability; random dynamical systems

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1. Introduction and main results

Parrondo's paradox is a well-known paradox in game theory that affirms that *a combination of losing strategies can become a winning strategy*, see [9, 11]. The paradox can be used to describe many situations away from game theory. In this work, we study the stability of fixed points and we shall establish the following analogy: attracting or repelling fixed points correspond to winning or losing strategies, respectively. Then, the question is the following: a fixed point that is attracting for f and g , can it be repelling for $f \circ g$? If so, we say the dynamical Parrondo's paradox arises.