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Highest weak focus order for trigonometric Liénard equations

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Abstract
Given a planar analytic differential equation with a critical point which is a weak focus of order k, it is well known that at most k limit cycles can bifurcate from it. Moreover, in case of analytic Liénard differential equations this order can be computed as one half of the multiplicity of an associated planar analytic map. By using this approach, we can give an upper bound of the maximum order of the weak focus of pure trigonometric Liénard equations only in terms of the degrees of the involved trigonometric polynomials. Our result extends to this trigonometric Liénard case a similar result known for polynomial Liénard equations.

Keywords Trigonometric Liénard equation · Weak focus · Cyclicity

Mathematics Subject Classification Primary 34C07 · Secondary 13H15 · 34C25 · 37C27

1 Introduction and main results

Recall that a critical point of a planar analytic vector field is called a *focus* if the eigenvalues of its linear approximation at the point are not real, i.e., $\alpha \pm \mathrm{i}\beta$, $\beta \neq 0$. Moreover, when $\alpha \neq 0$ the point is called a *strong focus* and, otherwise, it is called a *weak focus*. The complex Poincaré's normal form of its associated differential equation at this weak focus point is

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