Rendiconti del Circolo Matematico di Palermo Series 2 (2020) 69:771–785
https://doi.org/10.1007/s12215-019-00433-0

Linear type global centers of linear systems with cubic homogeneous nonlinearities

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Received: 11 October 2018 / Accepted: 19 July 2019 / Published online: 27 July 2019

Abstract
A center p of a differential system in $\mathbb{R}^2$ is global if $\mathbb{R}^2 \setminus \{p\}$ is filled of periodic orbits. It is known that a polynomial differential system of degree 2 has no global centers. Here we characterize the global centers of the differential systems

$$\dot{x} = ax + by + P_3(x, y), \quad \dot{y} = cx + dy + Q_3(x, y),$$

with P_3 and Q_3 homogeneous polynomials of degree 3, and such that the center has purely imaginary eigenvalues, i.e. a linear type center.

Keywords Center · Global center · Cubic polynomial differential system

Mathematics Subject Classification 34C05 · 34C07 · 34C08

1 Introduction and statement of the main results

The notion of center goes back to Poincaré and Dulac, see [6,10]. They defined a center for a vector field on the real plane as a singular point having a neighborhood filled of periodic orbits with the exception of the singular point. The problem of distinguishing when a monodromic singular point is a focus or a center, known as the focus-center problem started precisely with Poincaré and Dulac and is still active nowadays with many questions still unsolved. These

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