International Journal of Bifurcation and Chaos, Vol. 30, No. 1 (2020) 2050010 (12 pages)

DOI: 10.1142/S0218127420500108

# Nilpotent Global Centers of Linear Systems with Cubic Homogeneous Nonlinearities

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Received March 29, 2019

In this paper, we characterize the global nilpotent centers of polynomial differential systems of the linear form plus cubic homogeneous terms.

*Keywords*: Nilpotent center; cubic polynomial differential system; global center; degenerated hyperbolic sector.



## 1. Introduction and Statements of the Main Results

Poincaré [1951] and Dulac [1908] defined a *center* for a real planar vector field as a singular point whose neighborhood is filled with periodic orbits with the exception of the singular point. The so-called *focus-center problem*, which consists of distinguishing when a monodromic singular point is a focus or a center, started with these orbits but it is still very active with many open problems (see for instance [Algaba *et al.*, 2018a; Christopher & Li, 2007]).

If a real planar analytic system has a center at the origin, then after a linear change of variables and a rescaling of its independent variable, it can be written in one of the following three forms:

$$\dot{x} = -y + P(x, y), \quad \dot{y} = x + Q(x, y),$$

called a *nondegenerate center*;

$$\dot{x} = y + P(x, y), \quad \dot{y} = Q(x, y),$$

called a *nilpotent center*;

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y),$$

called a *degenerate center*, where $P(x, y)$ and $Q(x, y)$ are real analytic functions without constant and linear terms, defined in a neighborhood of the origin.